NUWC-NPT Technical Report 10,816
8 August 1997

# A Theoretical Control Study of the Biologically Inspired Maneuvering of a Small Vehicle Under a Free Surface Wave

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## PREFACE

This report was prepared under NUWC Division Newport Project No. 97WX30344, "Scientific Foundation of an Agile Vehicle;" principal investigator P. R. Bandyopadhyay (Code 8233). The sponsoring activity is Office of Naval Research (ONR), program managers Teresa McMullen (ONR) and S. C. Dickinson (NUWC).

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The technical reviewer for this report was James Dick (Code 8233).

**Reviewed and Approved: 8 August 1997**

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**Tactical Vehicle Systems Department**

# REPORT DOCUMENTATION PAGE

*Form Approved OMB No. 0704-0188*



| 1. AGENCY USE ONLY (Leave Blank) | 2. REPORT DATE | 3. REPORT TYPE AND DATES COVERED |
|---|---|---|
| | 8 August 1997 | Final |

4. TITLE AND SUBTITLE

A Theoretical Control Study of the Biologically Inspired Maneuvering of a Small Vehicle Under a Free Surface Wave

5. FUNDING NUMBERS

PR 97WX30344

6. AUTHOR(S)

Sahjendra N. Singh and Promode R. Bandyopadhyay

7. PERFORMING ORGANIZATION NAME(S) AND ADDRESS(ES)

Naval Undersea Warfare Center Division
1176 Howell Street
Newport, Rhode Island 02841-1708

8. PERFORMING ORGANIZATION REPORT NUMBER

TR 10,816

9. SPONSORING/MONITORING AGENCY NAME(S) AND ADDRESS(ES)

Office of Naval Research
800 N. Quincy St.
Arlington, Virginia 22217-5000

10. SPONSORING/MONITORING AGENCY REPORT NUMBER

11. SUPPLEMENTARY NOTES

12a. DISTRIBUTION/AVAILABILITY STATEMENT

Approved for public release; distribution is unlimited.

12b. DISTRIBUTION CODE

13. ABSTRACT (Maximum 200 words)

This report considers a theoretical control study of low-speed maneuvering of small underwater vehicles in the dive plane using dorsal and caudal fin-based control surfaces. The two dorsal fins are long and are actually mounted in the horizontal plane. The caudal fin is also horizontal and is akin to the fluke of a whale. Dorsal-like fins mounted on a flow aligned vehicle produce a normal force when they are cambered. Using such a device, depth control can be accomplished. A flapping foil device mounted at the end of the tailcone of the vehicle produces vehicle motion that is somewhat similar to the motion produced by the caudal fins of fish. The moment produced by the flapping foils is used here for pitch angle control. A continuous adaptive sliding mode control law is derived for depth control via the dorsal fins in the presence of surface waves. The flapping foils have periodic motion and they can produce only periodic forces. A discrete adaptive predictive control law is designed for varying the maximum tip excursion of the foils in each cycle for the pitch angle control and for the attenuation of disturbance caused by waves. The derivation of control laws requires only imprecise knowledge of the hydrodynamic parameters and large uncertainty in system parameters is allowed. In the closed-loop system, depth trajectory tracking and pitch angle control are accomplished using caudal and dorsal fin-based control surfaces in the presence of system parameter uncertainty and surface waves.

14. SUBJECT TERMS

Bio-Kinetics
Flapping Foils
Hydrodynamics
Hydromechanics
Oscillating Foils

15. NUMBER OF PAGES

20

16. PRICE CODE

| 17. SECURITY CLASSIFICATION OF REPORT | 18. SECURITY CLASSIFICATION OF THIS PAGE | 19. SECURITY CLASSIFICATION OF ABSTRACT | 20. LIMITATION OF ABSTRACT |
|---|---|---|---|
| Unclassified | Unclassified | Unclassified | SAR |

NSN 7540-01-280-5500

Standard Form 298 (Rev 2-89)
Prescribed by ANSI Std Z39-18
298-102

## TABLE OF CONTENTS

## LIST OF SYMBOLS*

| | |
|---|---|
| $A_1, A_2$ | Maximum cross-stream travel of a flap tip |
| $A_p, b_1, A_c, B_c, D_c$ | System matrices |
| $a_i, b_{ii}$ | System parameters |
| $u = U, w$ | Vehicle's forward and normal velocity |
| $q$ | Pitch rate |
| $\theta$ | Pitch angle |
| $z$ | Depth |
| $\delta$ | Camber |
| $Z_{\dot{q}}, Z_{\dot{w}}, Z_q, Z_w, Z_\delta$ | Coefficients used in representing normal force |
| $M_{\dot{q}}, M_{\dot{w}}, M_q, M_w$ | Coefficients used in representing moment |
| $x_B, z_B$ | Coordinates of CB |
| $x_g, z_g$ | Coordinates of CG |
| $C_d$ | Coefficient used in cross-flow integration |
| $m, W$ | Mass, weight of vehicle |
| $m_{p1}, m_{p2}, m_p$ | Moments produced by caudal fins |
| $m_d, f_d$ | Moment and force due to surface wave |
| $I_y$ | Moment of inertia |
| $\omega_f, \omega_o$ | Frequencies of oscillation of foils and of surface wave |
| $\alpha_1, \alpha_2, \alpha_o$ | Phase angles |
| $F_{io}, F_{ij}, M_{io}, M_{ij}$ | Flapping foil force and moment terms |
| $S_{t1}, S_{t2}$ | Strouhal numbers of foils |
| $U_c = (\delta, f_p, m_p)^T$ | Control vector |
| $\xi = (z, w, q, \theta)^T$ | State vector |
| $y_o = z$ | Output variable to be controlled |
| $y_r$ | Reference depth trajectory |
| $\zeta_r, \omega_r$ | Command generator parameters |
| $z^*$ | Target depth |
| $S$ | Switching surface |
| $e = (y_o - y_r)$ | Depth tracking error |
| $\lambda$ | Switching surface parameter |
| $\alpha, \Delta\alpha$ | Known and uncertain functions |
| $F_{d1}, F_{d2}$ | Amplitudes of sinusoidal force components due to surface wave |
| $\hat{F}_{d1}, \hat{F}_{d2}, \hat{\eta}$ | Estimates of parameters in control law |
| $L_1, L_2, L_3$ | Weighting parameters in the Lyapunov function |

* All variables nondimensionalized.

## LIST OF SYMBOLS (Cont'd)

| | |
|---|---|
| $V_o$ | Lyapunov function |
| $K$, $\mu$ | Gain, feedback parameter using in sliding mode control law |
| $e = (z - y_o)$ | Tracking error |
| $\theta^*$, $q^*$, $\omega^*$ | Equilibrium values |
| $T_p$ | Period |
| $q_a$ | Advance operator |
| $a_{cij}$, $B_{cj}$, $a_{fi}$, $B_{fi}$ | Elements used in discrete-time representation of dynamics |
| $\widetilde{\theta}$ | Pitch angle error |
| $J$ | Performance index |
| $\nu$, $\beta$ | Polynomials used for predicting pitch angle |
| $\nu_I$, $\beta_i$ | Coefficients of polynomials |
| $\hat{\nu}, \hat{\beta}$ | Estimated polynomials |
| $\hat{\nu}_i, \hat{\beta}_i$ | Estimates of $\nu_I$, $\beta_i$ |
| $\psi$, $\phi(k)$ | Regressor vectors |
| $p_o$ | Parameter vector |

# A THEORETICAL CONTROL STUDY OF THE BIOLOGICALLY INSPIRED MANEUVERING OF A SMALL VEHICLE UNDER A FREE SURFACE WAVE

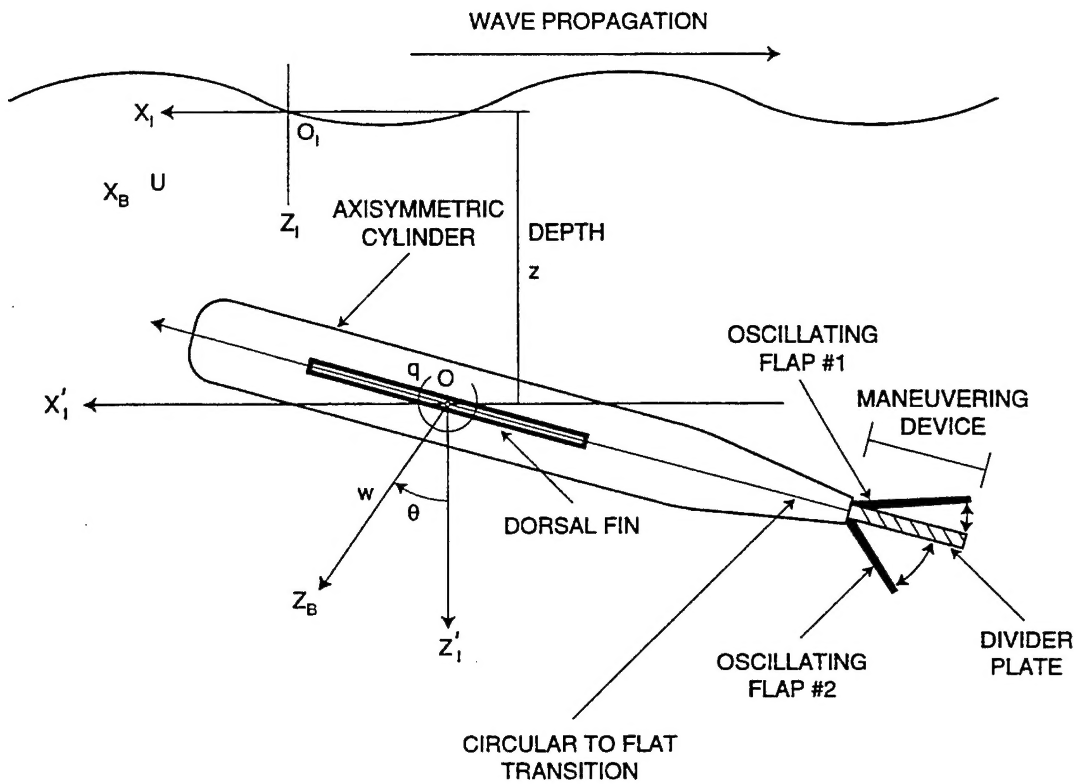
## INTRODUCTION

Biologically inspired maneuvering of man-made vehicles has the potential of being of value to the Navy. Aquatic animals have the ability to perform intricate maneuvers with great agility and, at the same time, very quietly. Fish, for example, have several configurations of fins including the dorsal and caudal fins. These fins provide a remarkable ability to fish for swift and complex maneuvers (Wu et al.[1] and Azuma[2]). Apparently, biologically-inspired dorsal and caudal fin-like control surfaces have a great potential for maneuvering small agile vehicles at low speed. However, application of these control surfaces to small undersea vehicles for quiet but agile maneuvering has remained unexplored.

Presently at the Naval Undersea Warfare Center (NUWC) Division, Newport, RI, considerable effort is in progress to study fish morphology and locomotion (Bandyopadhyay,[3] Bandyopadhyay and Donnelly,[4] and Bandyopadhyay et al.[5]). Experimental results conducted using several species of fish have provided interesting data for the design of control surfaces for low-speed maneuvering. Bandyopadhyay and co-researchers have designed caudal- and dorsal-like fins and studied the hydrodynamics of oscillating and cambering fins. The flow pattern and vortices formed have been recorded (Bandyopadhyay et al.[6]) in tests performed in tow tanks and water tunnels, and the forces and moments produced by the control surfaces have been measured. Related research to produce propulsive and lifting forces using flapping foil devices has been conducted by several authors.[7-12] However, as yet, control systems synthesis using caudal and dorsal fins has not been accomplished.

The contribution of the present research lies in the design of control systems for low-speed maneuvering of small undersea vehicles using dorsal- and caudal-like fins (figure 1). It is assumed that the hydrodynamic parameters of the vehicle are imprecisely known and surface wave-induced forces are constantly acting on the vehicle. Although the design approach can be extended to yaw control, in this study, only control in the dive plane is considered. Using the dorsal fin, a normal force is produced for depth control and flapping foils produce pitching moment for pitch angle regulation. For simplicity, it is assumed that the vehicle is equipped with a control mechanism that causes the vehicle to move forward with a uniform velocity. For the depth trajectory control, an adaptive sliding mode control law (Slotine and Li,[13] Utkin,[14] Narendra and Annaswamy[15]) is

designed for the continuous cambering of the dorsal fins in the presence of seawaves. The sliding mode control law is nonlinear and discontinuous in the state space and has an excellent insensitivity property with respect to disturbances and parameter variations.




where:

$X_I - Z_I$ = Inertial Coordinate System (Origin at the Calm Surface).

$X_I' - Z_I'$ = Translation of Inertial Frame (Origin at Geometrical Center).

$X_B - Z_B$ = Body Fixed Coordinate System.

(Note that the long dorsal fins are actually mounted in the horizontal plane. The caudal fins are also mounted in the horizontal plane and are akin to flukes in whales.)

***Figure 1. Schematic of the Maneuvering Devices (Dorsal and Caudal Fins) and Axisymmetric Cylinder***

The hydrodynamics of flapping foils is rather complex. Although design based on the continuous control of the angular velocity of the fins is more efficient, forces and moments produced by the caudal-like fins as functions of angular position and velocity is not well-understood. This study is limited to a periodic (sinusoidal) actuation of flapping foils. It is assumed that the foils have identical periods of oscillation that do not necessarily coincide with the period of the seawave. The amplitude and phase of force and moment acting on the vehicle caused by the disturbing wave is assumed to be unknown. Assuming that the pitch angle deviation is small, a linear discrete adaptive predictive control system (Goodwin and Sin[16]) is designed for the pitch angle control. In order to develop periodic moment, the maximum travel of the tips of the foils is adjusted periodically at the completion of the cycle. Interestingly, for the design of the pitch controller, it is seen that Strouhal numbers, which characterize the moment produced by the foils, are key control variables. In the closed-loop system using the dorsal and caudal fin controllers, depth control and pitch angle regulation in the dive plane are accomplished.

## MATHEMATICAL MODEL OF DIVE PLANE MOTION

Consider the vehicle motion in the dive (vertical) plane (figure 1). The heave and pitch equations of motion are described by coupled nonlinear differential equations. In a moving coordinate frame fixed at the vehicle's geometrical center, the dimensionless equations of motion for a neutrally buoyant vehicle are given by[17-19]

$$m\left(\dot{w} - uq - z_G q^2 - x_G \dot{q}\right) = z_{\dot{q}}\dot{q} + z_{\dot{w}}\dot{w} + z_q q + z_w w$$

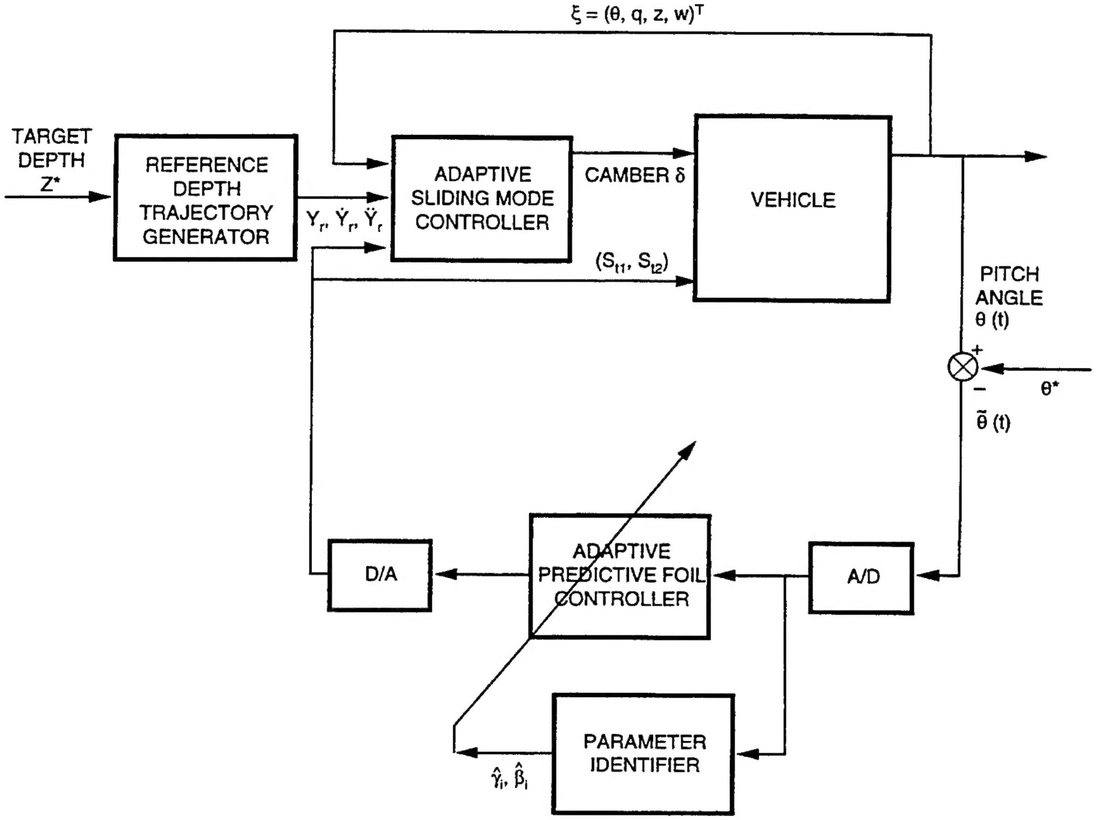
$$-C_D \int_{tail}^{nose} b(x)(w - xq)|w - xq|dx + z_\delta \delta + f_p + f_d,$$

$$I_y \dot{q} + m z_G(\dot{u} + wq) - m x_G(\dot{w} - uq) = M_{\dot{q}}\dot{q} + M_{\dot{w}}\dot{w} + M_q q \tag{1}$$

$$+M_w w + C_D \int_{tail}^{nose} x b(x)(w - xq)|w - xq|dx$$

$$-x_{GB} W \cos\theta - z_{GB} W \sin\theta + m_p + m_d ,$$

where $\dot{\theta} = q$, $\dot{z} = -u\sin\theta + w\cos\theta$, $x_{GB} = x_G - x_B$, $z_{GB} = z_G - z_B$, $\delta$ is the camber of the dorsal fins, $m_p = m_{p1} + m_{p2}$, $m_{pi}$ is the moment produced by the $i$th foil, $f_p$ is the net normal force produced by the flapping foils, and $m_d$ and $f_d$ are the force and moment acting on the vehicle caused by the surface wave. Here it is assumed that the forward speed is held steady ($u = U$) by a control mechanism. These nondimensionalized equations of motion (equation (1)) are obtained by dividing the original force and moment equations by $\frac{1}{2}\rho L^2 V^2$ and $\frac{1}{2}\rho L^3 V^2$ where $L$ and $V = U$ are the reference values for length and velocity, and the time is scaled by $(U/L)$. Thus $z_\delta$, $f_p$, and $m_p$ are the hydrodynamic coefficients of the vertical force and the pitching moment.




where $Z^*$ = Target Depth,
$\theta$ = Pitch Angle,
$\theta^*$ = Equilibrium Pitch Angle,
$S_{t1}$, $S_{t2}$ = Strouhal Numbers of Foils,
$A_1$, $A_2$ = Maximum Travel of Foils,
$\delta$ = Camber of Dorsal Fins.

***Figure 2. Closed-Loop System (Including the Caudal and Dorsal Fin Controllers)***

Bandyopadhyay et al.[20] have experimentally measured the forces and moments acting on winged bodies submerged in proximity of surface waves. The disturbance force and moment caused by surface waves are periodic, which can be expressed by a Fourier series. For simplicity in presentation, consider that $f_d$ and $m_d$ are well approximated by their fundamental components and are given by

$$
\begin{aligned}
f_d &= F_d \cos(\omega_o t + \alpha_o) \\
m_d &= M_d \cos(\omega_o t + \alpha_o),
\end{aligned}
\qquad (2)
$$

where $\omega_o$ is the fundamental frequency of the surface wave, $F_d$ and $M_d$ are amplitudes, and $\alpha_o$ is the phase angle.

The dorsal fin produces a normal force $(z_\delta \delta)$ proportional to the camber $\delta$ of the fins and can be continuously varied for the purpose of control. The forces and moments produced by the flapping foils are quite complex and depend on motion pattern (clapping and waving) as well as on the frequency of oscillation, maximum flapping angles, axis about foils oscillate, and the speed $U$. The choice of flapping parameters and the mode of oscillation can produce a variety of control forces and moments. Based on the experimental results and analysis, it had been shown by Bandyopadhyay and coworkers[3,4,6] that flapping foils produce periodic forces whose period is equal to the period of flapping. Therefore, their periodic forces can be expressed by a Fourier series, but are dominated by their fundamental components. Although the approach of this report can be generalized, for simplicity, it is assumed that the flapping foils produce forces and moments of the form

$$\begin{aligned} f_p &= F_{10}(S_{t1},\omega_f) + F_{20}(S_{t2},\omega_f) + F_{11}(S_{t1},\omega_f)\cos(\omega_f t + \alpha_1) \\ &\quad + F_{12}(S_{t2},\omega_f)\cos(\omega_f t + \alpha_2) \\ m_p &= M_{10}(S_{t1},\omega_f) + M_{20}(S_{t2},\omega_f) + M_{11}(S_{t1},\omega_f)\cos(\omega_f t + \alpha_1) \\ &\quad + M_{12}(t_{t2},\omega_f)\cos(\omega_f t + \alpha_2), \end{aligned} \tag{3}$$

where $S_{ti}$ is the Strouhal number defined as

$$S_{ti} = \left(\frac{fA_i}{U}\right), \qquad i = 1,2 \tag{4}$$

and is a dimensionless angular frequency parameter, $\omega_f$ is the frequency of oscillation, and $A_i$ is the maximum cross-stream travel of the flap tip. It is important to note that the Strouhal number of each foil is a key control variable that can be altered by the choice of frequency and the tip travel $A_i$ independently, and, thus, one can control the contribution of each foil in force generation for the purpose of control. Indeed, as shown by Bandyopadhyay,[3] by an analysis of a simplified two-dimensional momentum model, lateral steady forces produced by the two foils in clapping mode are

$$F_{10}(S_{t1}, f_1) = f_1 m_1 U_1 \sin(A_i/2), \tag{5}$$

and

$$F_{20}(S_{t2}, f_2) = -f_2 m_2 U_2 \sin(A_2/2), \tag{6}$$

where for the $i$th flap, $f_i$, is the frequency of oscillation, $m_i$ is the mass of water it affects, and $U_i$ is the velocity of water caused by the flapping action at an angle of $\tan^{-1}\left(\frac{A_i}{2C_h}\right)$ to the axial direction where $C_h$ is the chord. Thus, for $S_{t1} = S_{t2}$ and $A_1 = A_2$, $F_{10} + F_{20} = M_{10} + M_{20} = 0$ and by flapping, one produces pure sinusoidal forces of amplitudes $M_{11}$ and $M_{22}$, and if $S_{t1} \neq S_{t2}$, flapping action yields nonzero (positive or negative) constant force component $F_{10}(S_{t1}, f_1) + F_{20}(S_{t2}, f_2)$, as well as a time-varying periodic component.

For the purpose of control, in this study, it is assumed that the two foils are controlled independently and oscillate with the same frequency $\omega_f$, but the maximum travel of each tip $A_i$ is varied at the interval of $T_p$, the time period of oscillation of foils. A continuous change of $A_1$ and $A_2$ is not allowed here since the intention is to develop a periodic force by flapping, although such an imposed mode of oscillation does create a complex control design problem. Note that we are trying to imitate biolocomotion for slow speed maneuvers.

The problem of interest here is to design a control system for the independent control of depth ($z$) using dorsal fins and stabilize the pitch angle dynamics using flapping foils. This decomposition of the dive plane control problem simplifies the controller design. An adaptive sliding mode control system is designed for large magnitude depth ($z$) control using only translatory dynamics, and a discrete adaptive predictive controller is designed for pitch angle regulation separately based on the decoupled rotational dynamics of the pitch angle of the vehicle. A judicious choice of controller design is essential since the dorsal fins are continuously cambered, the parameters of oscillations of the foils can be altered only at the completion of the cycle of flapping at discrete, but uniformly distributed, instants of time.

The system (equation (1)) can be written in a vector form as

$$\begin{pmatrix} \dot{z} \\ \dot{w} \\ \dot{q} \\ \dot{\theta} \end{pmatrix} = \begin{bmatrix} -U\sin\theta + w\cos\theta \\ a_1 w + a_2 q + a_3(x_{GB}\cos\theta + z_{GB}\sin\theta) + a_4(w,q) + d_1 \\ a_5 w + a_6 q + a_7(x_{GB}\cos\theta + z_{GB}\sin\theta) + a_8(w,q) + d_2 \\ q \end{bmatrix} + \begin{bmatrix} 0 & 0 & 0 \\ B_{21} & B_{22} & B_{23} \\ B_{31} & B_{32} & B_{33} \\ 0 & 0 & 0 \end{bmatrix} \begin{bmatrix} \delta \\ f_p \\ m_p \end{bmatrix} \tag{7}$$

or

$$\dot{\xi} = A(\xi, d_1, d_2) + BU_c, \tag{8}$$

where $\xi = (z, w, q, \theta)^T \in R^4$ is the state vector ($T$ denotes transposition),

$U_c = (\delta, f_p, m_p)^T$ is the control vector,

$B = (Bij)$,

$$
\begin{aligned}
\Delta &= (m - z_{\dot{\omega}})(I_y - M_{\dot{q}}) - (mx_G + z_{\dot{q}})(mx_G + M_{\dot{w}}), \\
B_{22} &= (I_y - M_{\dot{q}})\Delta^{-1}, \\
B_{21} &= z_\delta B_{22} \\
B_{23} &= (mx_G + z_{\dot{q}})\Delta^{-1}, \\
B_{32} &= (M_{\dot{w}} + mx_G)\Delta^{-1}, \\
B_{31} &= B_{32} z_\delta, \\
B_{33} &= (m - z_{\dot{w}})\Delta^{-1}, \\
a_1 &= \left[(I_y - M_{\dot{q}})z_w + (mx_G + z_{\dot{q}})Mw\right]\Delta^{-1}, \\
a_2 &= \left[(I_y - M_{\dot{q}})(m + z_q) + (mx_G + z_{\dot{q}})(M_q - mx_G)\right]\Delta^{-1}, \\
a_3 &= -(mx_G + z_{\dot{q}})W\Delta^{-1}, \\
a_4 &= \left[(I_y - M_{\dot{q}})I_\omega + (mx_G + z_{\dot{q}})I_q + (I_y - M_{\dot{q}})mz_G q^2 - (mx_G + z_{\dot{q}})mz_G wq\right]\Delta^{-1}, \\
a_5 &= \left[(m - z_{\dot{\omega}})M_w + (mx_G + M_{\dot{w}})z_w\right]\Delta^{-1}, \\
a_6 &= \left[(m - z_{\dot{\omega}})(M_q - mx_G) + (mx_G + m_{\dot{w}})(m + z_q)\right]\Delta^{-1}, \\
a_7 &= -(m - z_{\dot{\omega}})W\Delta^{-1}, \\
a_8 &= \left[(m - z_{\dot{\omega}})I_q + (mx_G + M_{\dot{w}})I_w - (m - z_{\dot{\omega}})mz_G wq + (mx_G + M_{\dot{w}})mz_G q^2\right]\Delta^{-1}, \\
d_1 &= \left[(I_y - M_{\dot{q}})f_d + (mx_G + z_{\dot{q}})m_d\right]\Delta^{-1}, \\
d_2 &= \left[(m - z_{\dot{\omega}})m_d + (mx_G + M_{\dot{w}})f_d\right]\Delta^{-1},
\end{aligned}
\tag{9}
$$

and $I_w$, $I_q$ are the cross-flow integrals where

$$I_w = C_D \int_{tail}^{nose} b(x)(w - xq)|w - xq|dx,$$

and (10)

$$I_q = C_D \int_{tail}^{nose} xb(x)(w - xq)|w - xq|dx.$$

The matrices $A$ and $B$ are obtained by comparing equations (7) and (8).

For equation (8), we are interested in designing a dorsal fin control system for the depth control and a caudal fin control system for the pitch angle regulation. For the derivation of the control system, it is assumed that various hydrodynamic parameters and the amplitudes and phases of the force and moment induced by the surface wave are unknown.

## DORSAL FIN CONTROL SYSTEM

In this section, a dorsal fin control system is designed for depth control. Since depth ($z$) control is of interest, an output controlled variable

$$y_o = z \tag{11}$$

is associated with the system (equation (8)). Consider a reference trajectory, $y_r(t)$, generated by a second order command generator

$$\ddot{y}_r + 2\xi_r \, \omega_r \, \dot{y}_r + \omega_r^2 y_r = \omega_r^2 z^* , \tag{12}$$

where $z^*$ is the target depth coordinate, $\xi_r > 0$, and $\omega_r > 0$. The parameters $\xi_r$ and $\omega_r$ are properly chosen to obtain the desired command trajectories. The objective is to steer the vehicle using the dorsal fins so that $y_o = z(t)$ asymptomatically follows $y_r(t)$. As $y_o$ tends to $y_r(t)$, the vehicle attains the desired depth since $y_r$ converges to $z^*$.

For the derivation of a controller, an adaptive sliding mode control technique[13-15] is used and the sliding surface is defined as

$$S = \dot{e} + \lambda e , \tag{13}$$

where $\lambda > 0$ and $e = (y_o - y_r) = z - y_r$ is the tracking error. The sliding mode control law is a discontinuous function and switches whenever the trajectory crosses the surface $S =$ 0. In using a sliding mode control law, the evolution of trajectory proceeds in two phases. In the first phase, which is called the reaching phase, trajectory starting from any initial condition is attracted toward the switching surface. The subsequent motion takes place on the surface $S = 0$ and the trajectory essentially slides along the switching surface. This is the second phase of motion called the sliding phase.

Consider the motion during the sliding phase. During the period of sliding, one has $S(t) \equiv 0$, which implies from equation (13) that

$$\dot{e} + \lambda e = 0 . \tag{14}$$

Thus, during the sliding phase

$$e(t) = \overline{e}^{\lambda(t-t_s)} e(t_s), \tag{15}$$

where $t_s$ is the instant when the trajectory has reached the surface $S = 0$. According to equation (15), it follows that $e(t) \rightarrow 0$, that is, $z(t) \rightarrow z^*$ as $t \rightarrow \infty$ and the desired depth control is accomplished. Obviously, the motion during the sliding phase is insensitive to any disturbance input and uncertain parameter variations.

Now consider the design of a controller so that the trajectory beginning from any initial condition is attracted toward the switching surface. In obtaining a control law, differentiating $S(t)$ along the trajectory of the system (equation (8)) gives

$$
\begin{aligned}
\dot{S} &= \ddot{e} + \lambda\dot{e} \\
&= \ddot{z} - \ddot{z}_r + \lambda\dot{e} \\
&= -U\cos\theta\, q - q\sin\theta\, w + \cos\theta\,\dot{w} - \ddot{z}_r + \lambda\dot{e} \\
&= -(U\cos\theta + \sin\theta\, w)q + \cos\theta\, a_{r2}(\xi, d_1) + B_2 u_c - \ddot{z}_r + \lambda\dot{e},
\end{aligned}
\tag{16}
$$

where $a_{r2}(\xi,d_1)$ and $B_2$ are the second rows of vector $A$ and matrix $B$, respectively. Since depth control is to be executed by the dorsal fins, equation (16) is rewritten in the following form:

$$
\begin{aligned}
\dot{S} = \cos\theta B_{21}[\,&\alpha(\xi, f_p, m_p, t) + \Delta\alpha(\xi, d_1, f_p, m_p, t) + \eta^T\psi(\xi) \\
&+ F_{d1}\cos\omega_o t + F_{d2}\sin\omega_o t + \delta\,],
\end{aligned}
\tag{17}
$$

where $B_{21}^{-1}d_1 = F_{d1}\cos\omega_o t + F_{d2}\sin\omega_o t$. Here $\alpha$ and $\psi$ are known functions, but $\Delta\alpha$, the parameter vector $\eta$, the amplitudes $F_{d1}$ and $F_{d2}$, and $B_{21}$ are unknown. It is assumed that the sign of $B_{21}$ is known that $|\theta| \le \theta_m < \pi/2$. Without loss of generality, it is assumed that $B_{21} > 0$. The known functions $\alpha$ and $\psi$ are computed using the nominal set of values of various parameters of the system.

The camber $\delta$ of the dorsal fin is continuously varied to steer any trajectory toward the switching surface. Assuming that the frequency $\omega_o$ of the surface wave is known, a control law is now chosen as

$$
\begin{aligned}
\delta = &-\alpha(\xi, f_p, m_p, t) - \hat{\eta}^T\psi(\xi) - \hat{F}_{d1}\cos\omega_o t \\
&- \hat{F}_{d2}\sin\omega_o t - \mu S - K\,\mathrm{sgn}(S),
\end{aligned}
\tag{18}
$$

where $\mu > 0$, $\hat{\eta}$ and $\hat{F}_{di}$ are estimates of $\eta$ and $F_{di}$, respectively, and $K$ is a constant gain yet to be determined. Substituting control law equation (18) into equation (17), gives

$$
\begin{aligned}
\dot{S} = \cos\theta B_{21}[\,&\Delta\alpha(\xi, d_1, f_p, m_p, t) + \tilde{\eta}^T\psi(\xi) + \tilde{F}_{d1}\cos\omega_o t \\
&+ \tilde{F}_{d2}\sin\omega_o t - K\,\mathrm{sgn}\,S - \mu S\,],
\end{aligned}
\tag{19}
$$

where $\tilde{\eta} = \eta - \hat{\eta}$, and $\tilde{F}_{di} = F_{di} - \hat{F}_{di}$.

Now, adaptation laws for $\hat{\eta}$, $\hat{F}_{di}$, and gain $K$ must be chosen so that the surface $S$ becomes attractive to any trajectory of the system. In deriving the adaptation law, consider a Lyapunov function,

$$V_o\left(S,\tilde{\eta},\tilde{F}_{d1},\tilde{F}_{d2}\right)=\left((B_{21}\cos\theta)^{-1}S^2+\tilde{\eta}^T L_1\tilde{\eta}+\tilde{F}_{d1}^2 L_2+\tilde{F}_{d2}^2 L_3\right)/2, \tag{20}$$

where $L_1$ is any positive definitive symmetric matrix, $L_2>0$ and $L_3>0$. The derivative of $V_o$ is given by

$$\begin{aligned}\dot{V}_o &= S\left(\Delta\alpha+\tilde{\eta}^T\psi+\tilde{F}_{d1}\cos\omega_o t+\tilde{F}_{d2}\sin\omega_o t-K\,\mathrm{sgn}\,S\right)\\ &\quad-\mu S^2+\tilde{\eta}^T L_1\dot{\tilde{\eta}}+L_2\tilde{F}_{d1}\dot{\tilde{F}}_{d1}+L_3\tilde{F}_{d2}\dot{\tilde{F}}_{d2}.\end{aligned} \tag{21}$$

The function $V$ is a positive definite function of $S,\tilde{\eta},\tilde{F}_{d1},\tilde{F}_{d2}$ since $|\cos\theta|\geq\cos\theta_m$ and $V_o(0)=0$. In order to ensure that the surface $S=0$ is attractive, adaptation laws and $K$ are chosen so that $\dot{V}_o$ satisfies $\dot{V}_o\leq 0$.

In view of equation (21), one chooses the adaptation laws of the form

$$\begin{aligned}\dot{\hat{\eta}} &=-\dot{\tilde{\eta}}=L_1^{-1}\psi S,\\ \dot{\hat{F}}_{d1} &=-\dot{\tilde{F}}_{d1}=L_2^{-1}S\cos\omega_o t,\\ \dot{\hat{F}}_{d2} &=-\dot{\tilde{F}}_{d2}=L_3^{-1}S\sin\omega_o t,\end{aligned} \tag{22}$$

and the gain $K$ is chosen to satisfy

$$K=k_1\left(\xi,d_1,f_p,m_p\right)+\varepsilon, \tag{23}$$

where the function $k_1$ is a bound on the uncertain function satisfying

$$k_1\geq\left|\Delta\alpha\left(\xi,d_1,f_p,m_p,t\right)\right|. \tag{24}$$

Substituting adaptation law (22) in equation (21) now yields

$$\dot{V}_o\leq-\varepsilon|S|-\mu S^2\leq 0. \tag{25}$$

Since $\dot{V}_o\leq 0$, it follows that $S,\hat{\eta}$, and $\hat{F}_{di}$ are bounded. Furthermore, in view of equation (25), one has

$$\int_0^\infty\left(\mu S^2+\varepsilon|S|\right)dt\leq V(0)-V(\infty)<\infty, \tag{26}$$

which implies that $S$ is a square integrable function. Furthermore, boundedness of $S$ implies from equation (13) that $e, \dot{e}$ are bound. Assuming that the reference trajectory $y_r$, and $\dot{y}_r$ are bounded, it follows from boundedness of $\dot{e}$ that $w$ is bounded (in view of the computation for $\dot{z}$ in equation (7)). Then one concludes that $S(t) \to 0$, as $t \to \infty$, assuming that $\theta, q$ are bounded. This implies that the tracking error $(z - y_r) \to 0$ as $t \to \infty$. This completes the depth control system design.

The control law (equation (18)) includes a switching function that essentially compensates for the uncertain function $\Delta\alpha$ and the adaptive component depending on $\hat{\eta}$ compensates the linearly parameterized uncertain function $\eta^T \psi$. For countering the wave effect, sinusoidal components of frequency $\omega_o$, which are functions of the parameters $\hat{F}_{di}$, suffice. It is shown that parameter divergence may occur often in the adaptive system in the presence of modeling error (Sastry and Bodson[21]). There are several ways by which divergence of parameters $\left(\hat{\eta}, \hat{F}_{di}\right)$ can be avoided. Since the upper bounds on the hydrodynamic parameters can be assumed to be known, a projection method can be used to modify the adaptation law (equation (22)). According to the projection method,[21] the estimates $\hat{\beta}$ and $\hat{F}_{di}$ are set to their limiting values whenever the trajectory $(\hat{\eta}(t), \hat{F}_{di}(t))$ tries to escape its permissible range.

Assuming that error $y_r(t) \to z^*$, and $\dot{y}_r \to 0$, the control law (equation (18)) asymptotically decouples $(\theta, q)$ dynamics from the remaining variables. Thus, the residual dynamics of the system essentially describe the rotational pitch motion. This residual dynamics, when the motion is constrained so that the error $y - y_r = 0$, is called the zero-error dynamics (Slotine and Li[13]). For satisfactory performance in the closed-loop system, the state variables $\theta$ and $q$ associated with zero-error dynamics must be bounded. In the next section, control of pitch angle using flapping foils is considered.

Remark 1: In the derivation of the depth controller, it has been assumed that the frequency $\omega_o$ of the surface wave is known. However, it is pointed out that such an assumption is not essential and one can set

$$F_{di.} = \hat{F}_{di} = 0, \quad i = 1,2 \tag{27}$$

in equations (17) and (18) for control, but in this case one has a large value of $k_1$ satisfying $k_1 \geq |\Delta\alpha|$. This requires a large gain $k$ and, thus, larger magnitude of $\delta$ for control.

Synthesis of the discontinuous control law may lead to control chattering, which is undesirable. A way to avoid this phenomenon of chattering is to use an approximate but continuous control law instead of equation (18). This can be done easily by replacing the *sgn* function in equation (18) by a *sat* function where

$$\mathrm{sat}(S)=\begin{cases}(S/\varepsilon_1), & \text{if } |S|<\varepsilon_1\\ 1, & \text{if } S\ge\varepsilon_1\\ -1, & \text{if } S\le-\varepsilon_1\end{cases},\tag{28}$$

where $\varepsilon_1 > 0$ is the boundary layer thickness. Implementation of the approximate controller may lead to small terminal error, but this error tends to approach zero as $\varepsilon_1$ tends to 0.

## FLAPPING FOIL CONTROL OF PITCH DYNAMICS

In this section, control of rotational pitch dynamics (zero error dynamics) is considered. First, a discrete-time linear model for pitch control is obtained.

## DISCRETE-TIME PITCH DYNAMICS

Since the sliding mode controller asymptotically controls $z$ to $z^*$, the zero error dynamics is obtained from equation (1) by setting $\dot{e}=\dot{z}-\dot{y}_r=\dot{z}=0$. Also, when $e(t)=0$, $\dot{e}(t)=\dot{z}-\dot{y}_r=\dot{z}=0$, one has for small θ

$$w = U\theta.\tag{29}$$

It is assumed that the two foils oscillate with identical frequency $\omega_f$. The maximum travel $A_i$ of each foil-tip is independently controlled periodically at the interval of $T_p\left(=2\pi/\omega_f\right)$. This way the Strouhal numbers $S_{t1}$ and $S_{t2}$ of the two foils are independently controlled. The moments, $m_{pi}\left(S_{ti},\omega_f\right)$, and forces, $f_{pi}\left(S_{ti},\omega_f\right)$ $(i=1,2)$, generated by the flapping foils are nonlinear functions of the Strouhal numbers. Since $\omega_f$ is a constant, expanding $f_{pi}\left(S_{ti}\right)$ and $m_{pi}\left(S_{ti}\right)$ in the Taylor series about $S_{t1}=S_{t2}=S_t^*$, a constant, and neglecting higher order terms gives

$$\begin{aligned}B_{33}m_{pi}\left(S_{ti}\right)+B_{32}f_{pi} &\approx B_{33}\left[\frac{\partial M_{io}\left(S_t^*\right)}{\partial St_i}+\frac{\partial M_{1i}\left(S_t^*\right)}{\partial S_{ti}}\cos\left(\omega_f t+\alpha_i\right)\right]\widetilde{S}_{ti}\\ &+B_{32}\left[\frac{\partial F_{io}\left(S_t^*\right)}{\partial S_{ti}}+\frac{\partial F_{1i}}{\partial S_{ti}}\left(S_{ti}^*\right)\cos\left(\omega_f t+\alpha_i\right)\right]\triangleq b_{ii}(t)\widetilde{S}_{ti},\quad i=1,2,\end{aligned}\tag{30}$$

where $\widetilde{S}_{ti}=S_{ti}-S_t^*$.

Next, pitch angle must be regulated to $\theta^*$, a constant. Using equations (21), (22), (25), and (29), the pitch dynamics about ($\theta^*$, $q^*=0$) obtained from (7) are given by

$$\begin{pmatrix} \dot{q} \\ \dot{\tilde{\theta}} \end{pmatrix} = \begin{bmatrix} a_6 & a_5U + a_7(z_{GB}\cos\theta^* - x_{GB}\sin\theta^*) \\ 1 & 0 \end{bmatrix} \begin{bmatrix} q \\ \tilde{\theta} \end{bmatrix}$$

$$+ \begin{bmatrix} 1 \\ 0 \end{bmatrix} (b_{11}\tilde{S}_{t1} + b_{22}\tilde{S}_{t2} + D_2(t)) \tag{31}$$

where for small $\theta$

$$D_2 = d_2 + a_7 x_{GB}\cos\theta^* + a_8 + B_{31}\delta + a_5 U\theta^* + a_7 z_{GB}\sin\theta^*.$$

The system (31) can be written as

$$\dot{\tilde{x}} \triangleq A_p\tilde{x} + b_1[b_{11}\tilde{S}_{t1} + b_{22}\tilde{S}_{t2} + D_2(t)], \tag{32}$$

where

$$\tilde{x} = (q, \tilde{\theta})^T,\ \tilde{\theta} = \theta - \theta^*,\ A_p = \begin{bmatrix} a_b & a_5U + a_7(z_{GB}\cos\theta^* + x_{GB}\sin\theta^*) \\ 1 & 0 \end{bmatrix}, \text{ and}$$

$b_1 = [1,0]^T$.

The solution of equation (24) is given by[22]

$$\tilde{x}(t) = e^{A_p(t-t_o)}\tilde{x}(t_o) + \int_{t_o}^{t} e^{A_p(t-\tau)} b_1[b_{11}(\tau)\tilde{S}_{t1}(\tau) + b_{22}(\tau)\tilde{S}_{t2}(\tau) + D_2(\tau)]d\tau. \tag{33}$$

Since the control input $\tilde{S}_{ti}$ is to be implemented as a piecewise constant function changing at the interval $T_p$, a discrete-time model is obtained from equation (33) of the form

$$\tilde{x}(k+1) = A_c\tilde{x}(k) + B_c\tilde{S}_t(k) + D_c(k), \tag{34}$$

where $(k+1)$ denotes $(k+1)T_p$, $\tilde{S}_t(t) = \tilde{S}_t(k) = [\tilde{S}_{t1}(k), \tilde{S}_{t2}(k)]^T$ for $t\varepsilon[kT_p(k+1)T_p)$,

and

$$\begin{aligned} A_c &= e^{A_pT_p} \\ B_c &= \int_{kT_p}^{(k+1)T_p} e^{A_p[(k+1)T_p-\tau)b_1[\,b_{11}(\tau),b_{22}(\tau)]d\tau} \\ D_c(k) &= \int_{kT_p}^{(k+1)T_p} e^{A_p[(k+1)T_p-\tau]} b_1 D_2(\tau)d\tau. \end{aligned} \tag{35}$$

Let $A_c = (a_{cij})$, $B_c = [B_{c1}^T, B_{c2}^T]^T = (b_{cij})$ for $i, j = 1,2$, and $D_c(k) = (D_{c1}, D_{c2})^T$. Note that $B_c$ is a 2x2 constant matrix since integration in equation (35) is performed over one period $T_p$ and $\omega_f = 2\pi T_p$, but $D_c(k)$ depends on $kT_p$ due to the fact that $\omega_f \neq \omega_o$; that is, the flapping frequency differs from the frequency of the wave.

## AUTOREGRESSIVE MOVING AVERAGE MODEL

Next, a discrete adaptive predictive control technique is used for pitch control.[20] For this, an expression for the predicted value of $\widetilde{\theta}(k)$ is obtained and the advance operator $q_a$ is introduced and defined as

$$q_a z_s(k) = z_s(k+1) \tag{36}$$

for any discrete signal $z_s(k)$. Using equation (34) gives

$$q_a \widetilde{q}(k) = a_{c11}\widetilde{q}(k) + a_{c12}\widetilde{\theta}(k) + B_{c1}\widetilde{S}_t(k) + D_{c1}(k) \tag{37}$$

$$q_a \widetilde{\theta}(k) = a_{c21}\widetilde{q}(k) + a_{c22}\widetilde{\theta}(k) + B_{c2}\widetilde{S}_t(k) + D_{c2}(k). \tag{38}$$

Equation (37) gives

$$q_a^{-1}\widetilde{q}(k) = q_a^{-2}\left[a_{c11}\widetilde{q}(k) + a_{c12}\widetilde{\theta}(k) + B_{c1}\widetilde{S}_t(k) + D_{c1}(k)\right]. \tag{39}$$

Operating by $q_a^{-1}$ equation (38) gives

$$\widetilde{\theta}(k) = q_a^{-1}\left[a_{c21}\widetilde{q}(k) + a_{c22}\widetilde{\theta}(k) + B_{c2}\widetilde{S}_t(k) + D_{c2}(k)\right]. \tag{40}$$

Substituting for $q_a^{-1}\widetilde{q}(k)$ from equation (39) into equation (40) gives

$$\begin{aligned}\widetilde{\theta}(k) = {} & a_{c21}q_a^{-2}\left[a_{c11}\widetilde{q}(k) + a_{c12}\widetilde{\theta}(k) + B_{c1}\widetilde{S}_t(k) + D_{c1}(k)\right] \\ & + q_a^{-1}\left[a_{c22}\widetilde{\theta}(k) + B_{c2}\widetilde{S}_t(k) + D_{c2}(k)\right].\end{aligned} \tag{41}$$

Using equation (40), one obtains

$$q_a^{-2}\widetilde{q}(k) = \left[q_a^{-1}\widetilde{\theta}(k) - q_a^{-2}\left\{a_{c22}\widetilde{\theta}(k) + B_{c2}\widetilde{S}_t(k) + D_{c2}(k)\right\}\right]/a_{c21}, \tag{42}$$

which is substituted into equation (41) to yield

$$\tilde{\theta}(k) = a_{c11}\left[q_a^{-1}\tilde{\theta}(k) - q_a^{-2}\left\{a_{c22}\tilde{\theta}(k) + B_{c2}\tilde{S}_t(k) + D_{c2}(k)\right\}\right]$$

$$+ a_{c21}q_a^{-2}\left\{a_{c12}\tilde{\theta}(k) + B_{c1}\tilde{S}_t(k) + D_{c1}(k)\right\}$$

$$+ q_a^{-1}\left[a_{c22}\tilde{\theta}(k) + B_{c2}\tilde{S}_t(k) + D_{c2}(k)\right]. \tag{43}$$

Rearranging terms in equation (43), one has

$$\left[1 + (-a_{c11} - a_{c22})q_a^{-1} + (a_{c11}a_{c22} - a_{c21}a_{c12})q_a^{-2}\right]\tilde{\theta}(k)$$

$$= q_a^{-1}\left[B_{c2} + (a_{c21}B_{c1} - a_{c11}B_{c2})q_a^{-1}\right]\tilde{S}_t(k)$$

$$+ q_a^{-1}\left[D_{c2}(k) + a_{c21}q_a^{-1}D_{c1}(k) - a_{c11}q_a^{-1}D_{c2}(k)\right] \tag{44}$$

or

$$\left(1 + a_{f1}q_a^{-1} + a_{f2}q_a^{-2}\right)\tilde{\theta}(k) = q_a^{-1}\left(B_{f1} + B_{f2}q_a^{-1}\right)\tilde{S}_t(k) + q_a^{-1}a_{fd}(k), \tag{45}$$

where

$$\begin{aligned} a_{f1} &= -a_{c11} - a_{c22} \\ a_{f2} &= a_{c11}a_{c22} - a_{c21}a_{c12} \\ B_{f1} &= B_{c2} \\ B_{f2} &= a_{c21}B_{c1} - a_{c11}B_{c2} \\ a_{fd}(k) &= D_{c2}(k) + a_{c21}q_a^{-1}D_{c1}(k) - a_{c11}q_a^{-1}D_{c2}(k). \end{aligned} \tag{46}$$

The discrete-time model of equation (45) is called an autoregressive moving average (ARMA) model. The ARMA model can be expressed in an alternative predictor form using equation (46) rewritten as

$$\tilde{\theta}(k+1) = \left(-a_{f1} - a_{f2}q_a^{-1}\right)\tilde{\theta}(k) + \left(B_{f1} + B_{f2}q_a^{-1}\right)\tilde{S}_t(k) + a_{fd}(k). \tag{47}$$

This is a useful representation of the pitch dynamics. It is assumed that the parameters $a_{ft}$, $B_{ft}$, and the signal $a_{fd}(k)$ are unknown. For the regulation of $\theta(k)$, one can design predictive control laws if the estimates of the unknown parameters and $a_{fd}(k)$ are known.

For the derivation of a control law, it is assumed that

$$a_{fd}(k+1) \approx a_{fd}(k). \tag{48}$$

Note that if the wave frequency $\omega_o$ is equal to the frequency of flapping and if either $\dot{\delta}$ is small or $B_{31} \approx 0$, then $a_{fd}(k+1) = a_{fd}(k)$ for all $k$. In practice, it has been found

that the predictive control technique works well even when parameters vary slowly and the condition of equation (48) is violated.

Under the assumption of equation (48), subtracting $q_a^{-1}\widetilde{\theta}(k+1)$ from (47) gives

$$\widetilde{\theta}(k+1)=\left[-a_{f1}+\left(a_{f1}-a_{f2}\right)q_a^{-1}+a_{f2}q_a^{-2}\right]\widetilde{\theta}(k)$$

$$+\left[B_{f1}+\left(B_{f2}-B_{f1}\right)q_a^{-1}-B_{f2}q_a^{-2}\right]\widetilde{S}_t(k)\triangleq\upsilon(q^{-1})\widetilde{\theta}(k)+\beta(q^{-1})\widetilde{S}_t(k), \tag{49}$$

where

$$\begin{aligned}
&\upsilon_o=-a_{f1},\upsilon_1=a_{f1}-a_{f2},\upsilon_2=a_{f2},\beta_o=B_{f1},\beta_1=B_{f2}-B_{f1},\beta_2=-B_{f2},\\
&\upsilon\left(q_a^{-1}\right)=\upsilon_0+\upsilon_1 q_a^{-1}+\upsilon_2 q_a^{-2},\\
&\beta\left(q_a^{-1}\right)=\beta_0+q_a^{-1}\beta'\left(q_a^{-1}\right),\\
&\beta'\left(q_a^{-1}\right)=\beta_1+\beta_2\, q_a^{-1}.
\end{aligned} \tag{50}$$

## ADAPTIVE PITCH ANGLE CONTROL

Assuming that the parameters of equation (49) are known, now a weighted one-step ahead pitch control law is obtained. For this a suitable performance index of the form

$$J(k+1)=\frac{1}{2}\left[\widetilde{\theta}(k+1)-\theta_r^*(k+1)\right]^2+\frac{1}{2}\lambda_d\|S_t(k)\|^2 \tag{51}$$

is chosen, where $\lambda_d>0$ and $\theta_r^*(k)$ is a suitable reference trajectory to be followed by $\widetilde{\theta}(k)$. Note if $\theta_r^*(k)\rightarrow 0$, then $\theta(k)\rightarrow\theta^*$. By the choice of a suitable value of $\lambda_d$, a compromise between bringing $\widetilde{\theta}(k+1)$ to $\theta^*(k+1)$ and the amount of control effort expended is achieved.

Substituting $\theta(k+1)$ from (49) and (51) and for minimizing $J$ differentiating with respect to $S_t(k)$ gives

$$\beta_o^T\left[\upsilon\left(q_a^{-1}\right)\widetilde{\theta}(k)+\beta_o\widetilde{S}_t(k)+\beta'\left(q_a^{-1}\right)\widetilde{S}_t(k-1)-\theta_r^*(k+1)\right]+\lambda_d\widetilde{S}_t(k)=0. \tag{52}$$

Solving (52) gives the control law

$$\widetilde{S}_t(k)=\left(\lambda_d I+\beta_o^T\beta_o\right)^{-1}\beta_o^T\left[-\upsilon\left(q_a^{-1}\right)\widetilde{\theta}(k)-\beta'\left(q_a^{-1}\right)\widetilde{S}_t(k-1)+\theta_r^*(k+1)\right]. \tag{53}$$

Notice that the Strouhal number at the instant $kT_p$ depends on the present and past values of $\theta$ and the past values of input $S_t$, $\widetilde{S}_t$.

Since $\widetilde{S}_{ti}(k) \in [0, \widetilde{S}_{tim}]$ where $\widetilde{S}_{tim}$ is the same maximum allowed value of $\widetilde{S}_{ti}$, control input $\widetilde{S}_{ti}(k)$ given in (53) must be modified to meet the practical constraint on its magnitude.

A modified control law for pitch control is

$$\widetilde{S}_{ti}(k) = \begin{cases} \widetilde{S}_{ti}(k), & \text{if } 0 \le S'_{ti}(k) \le S_{tim} \\ 0, & \text{if } S'_{ti}(k) < 0 \\ \widetilde{S}_{tim}, & \text{if } S'_{ti}(k) > S_{tim} \end{cases}, \tag{54}$$

where $S'_t = (S'_{t1}, S'_{t2})^T$ denotes the expression in the right-hand side in equation (53).

## PARAMETER ESTIMATION

For synthesizing the control law, the parameters in equation (49) must be known. A practical solution to this problem is to obtain an estimate of these unknown parameters using an appropriate parameter identification technique. There are several kinds of algorithms based on the projection and the least square methods[20] that can be used to obtain the estimates of these unknown parameters $\beta_o, \upsilon_i$, and $\beta_i$ in equation (49). Equation (49) can be written as

$$\widetilde{\theta}(k+1) = \phi^T(k)\rho_o, \tag{55}$$

where

$$\phi(k) = \left[\left(1\, q_a^{-1}\, q_a^{-2}\right)\widetilde{\theta}(k), \left(1\, q_a^{-1}\, \upsilon_a^{-2}\right)S_t^T(k)\right]$$

$$\rho_o = \left[(\upsilon_o, \upsilon_1, \upsilon_2)^T, \beta_o^T, \beta_1^T, \beta_2^T\right].$$

Using a simple projection algorithm for parameter estimation, the estimate $\hat{\rho}_o$ of $\rho_o$ is obtained using an update law given by[20]

$$\begin{aligned} &\hat{\rho}_o(k) = \hat{\rho}_o(k-1) + \frac{a(k)\phi(k-1)}{c_1 + \phi^T(k-1)\phi(k-1)}\left[\widetilde{\theta}(k) - \phi^T(k-1)\hat{\rho}_o(k-1)\right] \\ &0 < a(k) < 2 \\ &c_1 > 0. \end{aligned} \tag{56}$$

Define

$$\begin{aligned} &\hat{\upsilon}(q_a^{-1}) = \hat{\upsilon}_o + \hat{\upsilon}_1 q_a^{-1} + \hat{\upsilon}_2 q_a^{-2} \\ &\hat{\beta}'(q_a^{-1}) = \hat{\beta}_1 + \hat{\beta}_2\, q_a^{-1}. \end{aligned} \tag{57}$$

Then the adaptive control law for pitch angle control is given by

$$\widetilde{S}_t(k) = \left(\lambda I + \hat{\beta}_o^T \hat{\beta}_o\right)^{-1}\left[-\hat{\upsilon}\left(q_a^{-1}\right)\widetilde{\theta}(k) - \hat{\beta}'\left(q_a^{-1}\right)\widetilde{S}_t(k-1) + \theta_r^*(k+1)\right]. \qquad (58)$$

This completes the design of flapping foil controller.

Now the adaptation law for adjusting the maximum travel of the tips of the two foils is easily computed using the definition of the Strouhal number and required adaptation scheme is given by

$$A_i(k+1) = \left[\frac{US_{ti}(k)}{\left(\omega_f / 2\pi\right)}\right], i = 1,2, \qquad (59)$$

where $S_{ti}(k) = \widetilde{S}_{ti}(k) + S_t^*$.

The complete closed-loop system is shown in figure 2.

## CONCLUSIONS

A theoretical study for the dive plane control system design for biologically inspired maneuvering of low speed, small undersea vehicles using dorsal and caudal fin-like control surfaces was considered. Normal force produced by the dorsal fin was used to control the depth of the vehicle and two flapping foils were used for the pitch angle control. An adaptive sliding mode control law was derived for the reference depth trajectory tracking. For the design of this control, a nonlinear vehicle model was considered for which the system parameters were assumed to be unknown, and it was assumed that sinusoidal disturbance force and moment are acting on the vehicle caused by surface waves. In the closed-loop system, including the sliding mode controller, depth control was accomplished and rotational pitch dynamics were asymptotically decoupled.

For the decoupled pitch dynamics, assuming that the pitch angle perturbations were small, a linear deterministic autoregressive model was derived. For the pitch angle control, the Strouhal numbers were chosen as key input variables. The Strouhal numbers of the two foils were periodically changed (at intervals of the time period of oscillations of the foils by altering the maximum tip travel). Both foils were oscillating at the same frequency. Using projection algorithms, the parameters of the pitch dynamics were identified. These estimated parameters were used to design an adaptive predictive control system. The adaptive predictive controller accomplished regulation of the pitch angle. Thus, in the complete closed-loop system, including the adaptive sliding mode and adaptive predictive controllers, dive plane control of the underwater vehicle can be accomplished in the presence of large parameter uncertainty and sea surface waves.

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